Session 1: Market Risk

Patrice Robin, Beirut, January 2015
Banking Risks

Session 1
- Credit Risk
- Market Risk
- Counterparty Credit Risk

Session 2
- Structural Interest Rate Risk
- Liquidity Risk
- ALM

Session 3
- Trading Book
- Banking Book
- Operational Risk
Market Risk Management Tools

Sensitivities (to risk factors)
  • Change in the price/mark-to-market of an instrument for a unit change in a pricing input (or risk factor)

Value-at-Risk
  • Probabilistic measure. How much could we lose on a position/portfolio at a given confidence interval over a given horizon?

Stress testing
  • Looks at impact of exceptional but plausible events
Agenda

1. Sensitivities: the case of forwards
2. Sensitivities: the case of bonds
3. Value at Risk (VaR)
FX forwards

Agreement to buy or sell a currency at a given future date and at predetermined price (the ‘Forward price’)

OTC product → documentation, credit risk

No payment up-front
No-arbitrage FX Forward

<table>
<thead>
<tr>
<th>EUR/USD 1,2340 spot (offer)</th>
<th>EURIBOR 3M 1,375%</th>
<th>3M $ LIBOR 0,27325%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tue 21/08/2012</td>
<td>Wed 21/11/2012</td>
<td></td>
</tr>
<tr>
<td>3m USD Libor</td>
<td>0.35000%</td>
<td>3M Euribor</td>
</tr>
</tbody>
</table>

Borrow 1.234 USD emprunté @ 0.35000% pour 92 jours 1.235104

Achat 1 EUR déposé @ 1.15000% pour 92 jours 1.002939

To be flat in both currencies at maturity USD 1,235104 must equal EUR1,002939; hence forward F= 1,231485
FX Forward: formula

\[ F = \frac{S \times \left(1 + r_{\text{quoted}} \times \frac{\text{days}}{\text{base}}\right)}{\left(1 + r_{\text{base}} \times \frac{\text{days}}{\text{base}}\right)} \]

S: Spot rate

\( r_{\text{quoted}} \): Libor in quoted currency

\( r_{\text{base}} \): Libor in base currency

days: number of days between spot and forward

base: 360 ou 365
Carry trade

Economics logic vs. Finance logic

Finance: no arbitrage principle

An increase in rates translate into depreciation for the currency (cf Forward formula)

But what do Central banks do when they wish to boost their currency? They increase rates… (Economics logic)

Carry trade: bet against the implied depreciation of the high rate currency
Carry trade: example

Borrow JPY1Md at 1% for 10 years
Buy AUD with the borrowed JPY, invest the AUD at 3.5% for 10 years
→ Carry = 2.5%

If at maturity AUD has not depreciated as expected by the forward formula then carry = profit
Exercise

EUR/SGD 1.6950
Euribor 3-month 0.15% (a/360)
SGD 3-month money-market rate 1.55% (a/365)

Q1. what is the non-arbitrage forward price?
Q2. I sell EUR 100m 3 months forward. What is the sensitivity of my mark-to-market to the 3 risk factors (use 1bp for rates and 1pip for FX)?
Forward Rate Agreements (FRAs)

A FRA is an agreement between two parties motivated by the wish either to hedge against or to speculate on a movement in future interest rates.

There is no commitment by either party to place or receive the underlying notional amount.
FRA: mechanics

- At T2 the buyer of the FRA receives:
  \[ \text{Notional} \times (\text{Libor} - \text{FRA rate}) \times \delta \]

- Generally the flow takes place at T1 and is therefore equal to:
  \[ \frac{\text{Notional} \times (\text{Libor} - \text{FRA rate}) \times \delta}{1 + \text{Libor} \times \delta} \]

![Diagram showing the mechanics of an FRA with 6M Libor Fixing, FRA traded at t = 0, and the formula for delta \( \delta = (T_2 - T_1) / 360 \) and the 6 months period between T1 and T2.]
FRA: Terminology

The underlying of the $3 \times 6$ FRA is the 3-month Libor ($=6-3$) fixing in 3 months’ time

A day is specified if fixing date is not exactly $x$ months from the trade date: The $6 \times 12$ 23rd FRA is the 6-month Libor that will fix on the 23rd of (current month + 6 months)

The ‘buyer’ or ‘payer’ or ‘taker’ of the FRA will have a positive payout when Libor fixes higher than the FRA rate

The ‘seller’ or ‘receiver’ or ‘giver’ of the FRA will have a positive payout when Libor fixes lower than the FRA rate
FRA: exercise

A corporate has $100m floating funding (indexed off 3m USD Libor) and believes that Libors will fix higher than the market expects in the near future. It therefore decides to fix the next fixing (Sep 14th) via a FRA. Today is July 25th.

1/ What is the sensitivity to the next Libor fixing?
2/ What FRA should be traded and is the corporate taking or giving the FRA?
3/ Assume the FRA was traded at 0.65% on July 25th and 3m Libor fixes at 0.71% on Sep 14th, what is the settlement amount, who pays it and when is it normally paid?
Agenda

1. Sensitivities: the case of forwards
2. Sensitivities: the case of bonds
3. Value at Risk (VaR)
Pricing a Bond - YTM

(Dirty) market price = 104.65

YTM is the discount rate such that the present value of the bond flows equals the bond dirty price

\[ PV = \sum_{i=1}^{n} \frac{C}{(1 + YTM)^i} + \frac{100}{(1 + YTM)^n} \]

N denotes the number of payments
YTM is such that the PV of the flows equals the market price of the bond
Measuring the price risk

Two important aspects of the price yield relationship

- Slope
- Non-linearity

These are measured using

- Modified Duration
- Convexity
Duration: A simple concept, used to compare bonds with different characteristics (coupon rate, frequency, maturity, ytm)

We distinguish between MacCaulay Duration and Modified Duration
MacCaulay Duration: PV weighted average life of a bond

\[
MacCaulay\,\text{Duration (years)} = \frac{\sum_{t=1}^{n} \frac{CF_t \cdot t}{(1 + r/m)^t}}{(DP \cdot m)}
\]

Where:
- \( n \) = number of coupons
- \( r \) = Quoted Yield Rate
- \( m \) = Compounding Periods per annum
- \( DP \) = dirty price of the bond
Modified Duration is a linear measure of the price / yield relationship at any point, for a 100bp shift in ytm

\[
\text{Modified Duration} = \frac{\text{MacCaulay Duration}}{(1 + (r / m))}
\]

Where:
- \( r \) = Quoted Yield Rate
- \( m \) = Compounding Periods per annum

Exercise: Compute the MacCaulay and Modified Durations for a 10y bond with 6% semi coupons. Assume current dirty price=100
## Duration

YTM 3.00% semi-annual

<table>
<thead>
<tr>
<th>t</th>
<th>Sum Flow</th>
<th>PV</th>
<th>100</th>
<th>PV*t</th>
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<td>2.91</td>
<td>1532.38</td>
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<tr>
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<td>1.71</td>
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<td>32.51</td>
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<td>20</td>
<td>103</td>
<td>57.03</td>
<td></td>
<td>1140.57</td>
</tr>
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</table>

**Duration**: 7.6619

**MD**: 7.4387
Duration

The (MacCaulay) Duration of a zero-coupon bond (ZCB) is equal to its maturity whilst any coupon-bearing structure has a duration which is shorter than maturity

→ Steepest price/yield relationship for ZCB

If a continuous ytm is used, the Modified Duration equals the MacCaulay Duration
Dollar Duration (Bloomberg call it ‘RISK’)

- If Modified Duration is a measure of the expected % price change per 100bps change in yield.
- Then the measure can be very simply converted into units of absolute currency by:

\[
\text{Dollar Duration} = \frac{\text{Modified Duration}}{100} \times \text{Dirty Price}
\]

- NB: Same concept as DV01 except that DV01 is normally computed using 1bp rather than 100bps
Convexity

- Convexity
  - The rate of change in Modified Duration with respect to change in yield. (Second Derivative of Price/Yield)

\[
Convexity = \frac{P^+ + P^- - 2P_0}{P_0 \times (\Delta r)^2 \times 100}
\]

- $P^+$ bond price on a 100bp shift down
- $P^-$ bond price on a 100bp shift up
- $P_0$ bond price at current ytm
- $\Delta r$ 1% yield shift
Convexity

- Two Rules of Convexity
  - For two bonds (portfolios) with the same Duration, the higher coupon bond (more time dispersed cash flows) will be more convex.
  - For two bonds with the same Maturity the lower coupon bond will be more convex

Exercise: Compute the convexity of the bond used in the previous exercise
Convexity

- Price / Yield Line
- Dollar Duration Line

Convexity

Price

Yield
Using Duration and Convexity

We can use the Modified Duration and Convexity to estimate the change in the dirty price for a given change in yield

$$\Delta DP = DP \times \left( -MD \times \Delta YTM + \frac{Cvexity \times \Delta YTM^2 \times 100}{2} \right)$$

DP: Dirty Price of the bond
In our example, the estimated change in DP for a 1% increase in ytm is equal to:

\[
\Delta DP = 100 \times \left( -7.4387 \times 1\% + \frac{0.6884 \times 1\%^2 \times 100}{2} \right)
\]

i.e. \( \Delta DP = -7.09455 \)

Compared to 7.106 actual change
Analogy with option Greeks

An option is a non-linear function of the underlying asset price (amongst other factors).

Delta denotes the sensitivity of the option price to changes in the price of the underlying asset (first derivative of the option price to the price of the underlying) → similar concept to Modified Duration.

Gamma denotes the sensitivity of Delta to changes in the underlying price (second derivative of the option price with respect to the underlying price) → similar concept to Convexity.
Callable bonds

In a callable bond, the issuer may trigger early redemption of the bond at a predetermined price. The bond will be called when the bondholder wants it least, giving rise to negative convexity.
Puttable bonds

In a puttable bond, the holder has the option to sell the bond back to the issuer at a predetermined price. What will it do to the convexity of the bonds?
Z-spreads (to Treasuries)

- The correct way to price a bond is to discount each flow with its associated zero rate:

\[ PV = \sum_{i=1}^{n} \frac{C}{(1+Z_i)^i} + \frac{100}{(1+Z_n)^n} \]

- For corporate bonds, the Z-spread is the (parallel) spread to be applied to the Treasury zero curve in order that the PV of all the flows equals the market price of the bond:

\[ PV = \sum_{i=1}^{n} \frac{C}{(1+Z_i+s)^i} + \frac{100}{(1+Z_n+s)^n} \]
Option-adjusted spreads

The Z-spread methodology is meaningless when applied to bonds with embedded optionality as the value of the option is ignored.

The OAS should be thought of as a Z-spread that has been adjusted for the value of the derivative.

A cancellable bond is equivalent to a package of 2 securities:

- A ‘stripped’ bond: identical bond (to the callable bond) stripped of its embedded call
- A call option
Option-adjusted spreads

Methodological steps:

• The callable bond = ‘stripped bond’ - call value
• The value of the call is computed using standard option valuation methods
• We now have a value for the stripped bond=callable bond price + option value
• The OAS is the spread to be applied to the zero curve so that the PV of the flows of the stripped bond is equal to its value computed above
Case Study

Consider the following bonds:

All annual 30/360 unadjusted

<table>
<thead>
<tr>
<th>T</th>
<th>Par</th>
<th>MP</th>
<th>Zero</th>
<th>Df</th>
<th>YTM</th>
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<tr>
<td>1</td>
<td>4%</td>
<td>101</td>
<td>2,9703%</td>
<td>0,97115</td>
<td>2,9703%</td>
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<tr>
<td>2</td>
<td>3%</td>
<td>99</td>
<td>3,5350%</td>
<td>0,93288</td>
<td>3,5266%</td>
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<tr>
<td>3</td>
<td>5,50%</td>
<td>103</td>
<td>4,4704%</td>
<td>0,87704</td>
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<tr>
<td>4</td>
<td>4%</td>
<td>97</td>
<td>4,9037%</td>
<td>0,82573</td>
<td>4,8430%</td>
</tr>
</tbody>
</table>

ABC corp. has a 4Y 8% bond (annual bond basis, coupon just paid) trading at 101
Case Study

1. What is the modified duration and convexity of the bond?

2. Use the answers from 1. to compute the expected price movement should the ytm decrease by 2% (using the formula slide 28). Compare your answer to a full reprice of the bond.

3. What is the yield spread over the 4Y Treasury?

4. What is the Z-spread of the bond?
Agenda

1. Sensitivities: the case of forwards
2. Sensitivities: the case of bonds
3. Value at Risk (VaR)
Value-at-Risk (VaR) Definition

How much can we lose with probability $p$ over a given time horizon $T$?

- Risk measured in percentiles
- Risk in ‘normal’ market conditions
- Risk factors identification
Value at Risk

Current value of holding

Probability distribution

95% VaR

P/L

5%
3 main types of implementation of VaR:

- Historical Simulation (HistSim) VaR
- Parametric VaR (or Analytical VaR)
- Monte Carlo VaR
Historical Simulation VaR

- Uses historical data to estimate potential future movements in market data
  - Pricing of portfolio given current data
  - Apply historical return to that valuation
  - Compute P/L and distribution
  - Take percentile
- Assumption:
  - Historical data adequate to model future movements
Historical Simulation VaR

- **Advantages:**
  - No need for assumptions with respect to probability distributions of risk factors
  - No need to assume correlation numbers: Correlations imbedded in the data

- **Disadvantage:**
  - Little reactivity in stressed market
  - How good is the past to predict the future?
HistSim VaR: Worked example

A USD-based investor holds 10,000 BNPP shares

Today is Jan 9th: BNPP share price = EUR45.97, EUR/USD 1.3125

Holding value = USD603,356

We compute the 1-day 95% VaR using the Historical simulation method, with one year of data
# Logreturns: basic statistics

**255 business days**

<p>| | |</p>
<table>
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<tbody>
<tr>
<td>min return</td>
<td>-7.958%</td>
</tr>
<tr>
<td>max return</td>
<td>11.044%</td>
</tr>
<tr>
<td>number of positive returns</td>
<td>143</td>
</tr>
<tr>
<td>number of negative returns</td>
<td>112</td>
</tr>
<tr>
<td>number of zero returns</td>
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Histogram of Logreturns

<table>
<thead>
<tr>
<th>Bin</th>
<th>Frequency</th>
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<tbody>
<tr>
<td>-11%</td>
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<td>More</td>
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HistSim VaR: worked example

255 observations, 95% Confidence Interval

VaR = 13th largest loss = -4.8458%

Holding value = USD603,356

VaR = 603,356 * 4.8458% = USD29,237
Expected Shortfall

Average of losses above a given confidence interval

Also called Expected Tail Loss (ETL) or Conditional VaR

In our example:

\[ \text{VaR} = -4.8458\% \ (13\text{th largest loss}) \]

Expected shortfall = average of the 12 losses higher than VaR

Expected shortfall = -6.1154\%
Parametric VaR

Requires joint probability distributions of the risk parameters returns

- Model joint distributions
- Deduct parameters from Variance-covariance matrix (obtained using market or historical data)
- Deduct volatility of portfolio value
- Take percentile

Assumes that Risk factor returns are normally distributed
Parametric VaR

Main advantage is speed of computation

Main drawback is the difficulty in taking into account optional positions
Parametric VaR (or analytical)

- Needs to be defined:
  - Risk factors identification
  - Sensitivities to these risk factors
  - Standard deviations of these risk factors
  - Variance-covariance matrix of these factors
Volatility of logreturns (computed historically over our one-year data) = 2.90%

Confidence Interval = 95% (1.645 standard deviations)

VaR = 1.645 * 2.90% = 4.77%

VaR = 603,356 * 4.77% = USD28,780

What if we wanted to use market data (instead of historical statistics)? What are the issues?
Correlation

Correlation coefficient between \(x \) & \(y\)

\[
\rho_{xy} = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y},
\]

covariance between \(x \) & \(y\)

\[
\text{cov}(x, y) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})
\]

Volatility of \(x\)

\[
\sigma_{xy} = \sqrt{w^2 \sigma_x^2 + (1-w)^2 \sigma_y^2 + 2w(1-w)\rho_{xy} \sigma_x \sigma_y}
\]

Standard deviation (volatility) of a two-asset portfolio

• \(w\) is the percentage of asset \(x\) in the portfolio
• \(1 - w\) is the percentage of asset \(y\) in the portfolio
Case study: Parametric & Historical VaR, Expected Shortfall
Parametric VaR: multi factor

VaR is computed for each risk factor

The correlation matrix must be specified to compute the portfolio VaR

$$\text{VaR}_{\text{portfolio}} = \sqrt{\sum_i \sum_j \text{VaR}_i \times \rho_{ij} \times \text{VaR}_j}$$

The difference between the portfolio VaR and the sum of the individual VaRs shows the impact of diversification

Example: VaR1=15,000; VaR2=20,000; correlation=0.3

VaR_{portfolio}=28,373

Diversification effect =of 6,627
2-factor Portfolio VaR as a function of the correlation input
Lognormality?

The chart shows a histogram of historical returns for a period from 6/27/11 to 6/26/12. The index is HRH with a delay of 0 days. The mean return is 0.143% and the sigma is 1.4883%. The Chi2/Ndf is 58.37/17 with a CL of 1.0. The normality test indicates that the data is not normally distributed.
Lognormality?
Lognormality?
Monte Carlo VaR

• Iterative Process:
  – Value the portfolio with current data
  – Draw a random sample of the joint probability distribution of market risk factors
  – Deduct the new potential value for market data
  – Reprice portfolio with that new data
  – Compute P&L
  – Repeat steps 2 to 5 a high number of times to estimate the P/L probability distribution
  – Take percentile
Monte Carlo VaR

• Assumption:
  – Choice of probability distribution

• Advantage:
  – Allows integration of options and exotics (e.g. path-dependent)

• Disadvantage:
  – Needs very high computational power
Monte Carlo VaR

Simulate the path of the asset price during the life of the option a high number of times

Deduct distribution of future asset value and take percentile

- $S_t$: underlying asset price at time $t$
- $\delta t$: small increment in time
- $r$, $q$: interest rates and dividends
- $\sigma$: volatility
- $\phi$: randomly generated factor

\[
S_{t+\delta t} = S_t \times \exp\left(\left[r - q - \frac{\sigma^2}{2}\right] \delta t + \sigma \sqrt{\delta t} \phi\right)
\]
Monte Carlo VaR
Regulatory VaR

- Internal VaR: 95% 1-day (1.645 standard deviations)
- Regulatory VaR: 99% 10-day (2.326 standard deviations)

$$\text{Regulatory VaR} = \frac{2.326}{1.645} \times \text{Internal VaR} \times \sqrt{10} \times \text{Multiplier}$$
VaR parameters – FT articles Oct 2012

http://www.ft.com/intl/cms/s/0/12b31c9a-1936-11e2-9b3e-00144feabdc0.html#axzz29jaRjUGx

http://ftalphaville.ft.com/2012/10/18/1218491/a-tale-of-two-vars/
Problems with VaR

- Extreme events far more common than assumed by Gaussian models (“Fat tails”, leptokurtosis)
- Relies exclusively on quantile hence disregards magnitude or distribution of losses beyond VaR
- VaR is not subadditive,
  i.e. if P1 and P2 are two portfolios, the VaR of the combined portfolio \( \text{VaR}(P1+P2) \) is not always inferior to the sum of VaRs \( \text{VaR}(P1) + \text{VaR}(P2) \)
- Expected Shortfall, however, is subadditive, and looks at the magnitude of the losses beyond the VaR level
Bank has 2 projects with same profile:
2% probability of $10m loss
98% probability of $1m loss
The 97.5% VaR on each project (separately) is $1m

Let’s have a look at the VaR on the 2 projects combined:
0.04% proba of $20m loss (2%*2%)
3.92% proba of $11m loss (2*2%*98%)
96.04% proba of $2m loss (98%*98%)
The 97.5% VaR is $11m > Sum of VaRs ($2m)
Using a non-subadditive technique to measure risk may:
- Lead to overly concentrated portfolios
- Lead banks to break up into smaller subsidiaries to reduce capital requirements
Recall the previous example. The loss distribution for a single project:
2% probability of $10m loss
98% probability of $1m loss

Expected Shortfall (97.5% CI) =
\[ 20\% \times 1m + 80\% \times 10m = 8.2m \]

The loss distribution for the combined projects:
0.04% proba of $20m loss (2\% \times 2\%)
3.92% proba of $11m loss (2 \times 2\% \times 98\%)
96.04% proba of $2m loss (98\% \times 98\%)

Expected Shortfall (97.5% CI) =
\[ 0.04\% \times 20m + 2.46\% \times 11m = 11.144m \]
Expected Shortfall (97.5% CI) = $11.144m

Sum of Expected Shortfalls = $16.4m ($8.2m * 2)

Expected shortfall is subadditive (when VaR was not)
Expected Shortfall looks at tail of distribution and is subadditive

But…

Difficult to backtest